## INTRODUCTION

Aerials are converters of electromagnetic energy. Transmitting aerial is meant for transformation of the electromagnetic waves (EMW) energy connected to directing systems, to the EMW energy in a free space and its radiation in given directions. The reception aerial performs inverse functions. Thus, each aerial carries out two functions: transforms one kind of electromagnetic energy to another; provides radiation of electromagnetic waves in certain directions, or reception of EMW, coming with certain directions.

The aerial properties follow from their definition:

1. Transformation of energy. If an input of transmitting aerial is fed by the waves connected to the directing system, which energy can be characterized by voltage $U$ and current strength $I$, we receive electromagnetic waves in free space on an output, energy of which is characterized by intensity of the electric field $E$ and intensity of the magnetic field $H$ (Fig. 0.1*(a)*).

2. Convertibility of the aerial. If to an output of the same aerial to bring energy of freely extending EMW on an input, we receive energy of the electromagnetic waves connected to the directing system (Fig.0.1*(b)*).

3. Energy directivity. Maintenance of main directions of radiation and reception in space enables to concentrate the radiation energy in the given solid angles, that allows to reduce the total radiated power.

Feeding devices are directing systems, that serve to transfer the EMW energy from the generator to the aerial and from the aerial to the receiver.

Classification of aerials by the principle of action is the most widely used. Classification by the form of current-

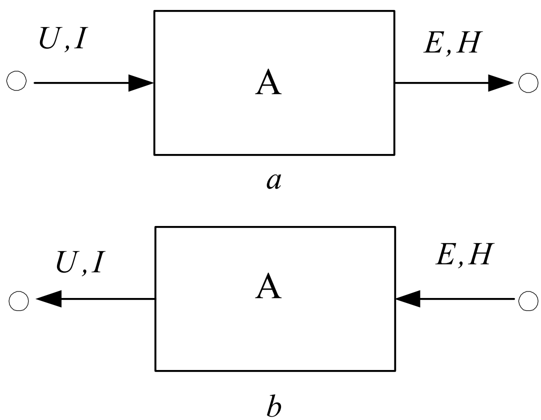


Fig. 0.1

carrying surfaces is close to it. Quite often aerials are classified by frequency ranges.

First, aerials are divided into active and passive. Passive aerials are further subdivided into two big classes: linear aerials and aperture aerials.

Any radiating system of a small (in comparison with the length) cross size, along which longitudinal axis the alternating current flows, are referred to as the linear aerials. The diameter of the section of the linear aerials is much less than the wavelength.

The aperture aerials are characterized by some surface through which all flux of electromagnetic energy, radiated or received, is propagated. This surface is referred to as the aperture surface. The aperture sizes considerably exceed the wavelength. Among these are the reflector-type antennas, horn, the open end of a waveguide, etc.

Antenna arrays consist of individual radiators: linear or aperture. They are used in the frequency ranges 3 MHz – 30 GHz. There may be linear, planar, circular, spherical and conform antenna arrays.

# 1. DETERMINATION OF THE RADIATION FIELD ON THE GIVEN SOURCES

The research of aerials in a mode of radiation is relatively simplier, than the research of aerials in a mode of electromagnetic waves reception. For the aerial, which properties in a mode of radiation are known, all parameters at work in a mode of reception may be found on the basis of the principle of reciprocity. Therefore, the theory of aerials is primarily studied as the theory of transmitting aerials.

## 1.1. Basic problems of the aerial theory

In the aerial theory the direct and inverse problems are considered to be the basic ones. The direct problem is to determine the radiation field from given to the aerial and its current. The essence of the inverse problem is to find or synthesize the aerial and to determine the way of its feed by the given distribution of the radiation field .

Let us consider the direct problem of the aerial theory in detail. The design and the form of the aerial, electric and magnetic properties of materials of which it is made, are considered to be known, as well as an arrangement of sources of external currents and charges. At the first stage of decision of the direct problem it is necessary to find out the distribution of currents, charges or tangential components of the field on the surface of the aerial. At the second stage from predetermined distribution of sources on surfaces of the aerial the electromagnetic field in space is found and parameters of the aerial are calculated. The second stage is known as the external problem of the aerial theory.

Using the second Maxwell's equation in the complex form

$$rot\dot{E} = -i\omega\mu_a\dot{H} \ ,$$

after substitution

$$\vec{\dot{H}} = \frac{1}{\mu_a} rot\vec{\dot{A}} \qquad (1.1)$$

we obtain

$$rot\left(\vec{\dot{E}} + i\omega\vec{\dot{A}}\right) = 0 ,$$

where $\dot{E}, \dot{H}$ are electric and magnetic field strengths, respectively; $\mu_a$ is the magnetic permittivity; $\omega$ is the pulsatance; $\dot{A}$ is the vector potential of the electromagnetic field.

Taking into account that the vector $\dot{E} + i\omega\dot{A}$ creates the potential field, some scalar function (potential) with the gradient exists

$$\dot{E} + i\omega\dot{A} = -grad\dot{U}$$

or

$$\dot{E} = -i\omega\dot{A} - grad\dot{U} \ . \qquad (1.2)$$

Substituting $\dot{E}, \dot{H}$ from (1.1), (1.2), respectively, in the first Maxwell equation and using the Lorenz gauge transformation

$$div\dot{A} + i\omega\dot{\varepsilon}_a\mu_a\dot{U} = 0 ,$$

we obtain the non-homogeneous wave equation for the vector potential

$$\nabla^2\dot{A} + \dot{\gamma}^2\dot{A} = -\mu_a j_{ex} , \qquad (1.3)$$

where $\nabla$ is the differential Hamilton's operator; $\dot{\gamma} = \omega\sqrt{\dot{\varepsilon}_a\mu_a}$ is the propagation factor; $j_{ex}$ is the density of external conduction current.

It is known from electrodynamics that solution of (1.3) is

$$\dot{\vec{A}} = \frac{\mu_a}{4\pi} \int_V \frac{\dot{j}_{ex} e^{-i\dot{\gamma} r}}{r} dV ,$$

(1.4)

where $r$ is the distance between the volume element $dV$ and the reference point, in which the vector potential value is determined; $V$ is the volume of the external current density.

Finally, for the electric intensity

$$\dot{\vec{E}} = \frac{1}{i\omega\dot{\varepsilon}_a} rot\dot{\vec{H}} = \frac{1}{i\omega\dot{\varepsilon}_a \mu_a} rotrot\dot{\vec{A}} .$$

(1.5)

The obtained field distribution should satisfy the radiation condition and boundary conditions. Distribution of currents, charges, tangential components of electromagnetic field for many antennas can be set on the basis of certain physical reasons.

### 1.2. Regulations of geometrical optics

When studying local - flat electromagnetic waves, methods of geometrical optics are often used, that simplifies a technique of research. The front of the local - flat waves in the considered limited area of space is close to the flat one.

The basis for the geometrical optics is four laws, established empirically: the law of the rectilinear propagation of radio waves; the law of independence of beam tubes, the law of reflection and the law of refraction. In geometrical optics the propagation of electromagnetic waves is investigated without taking into account the nature of wave processes by means of geometrical relations.

The trajectory, along which the electromagnetic energy is transferred, is known as the ray. From the point of view of geometrical optics it is supposed, that inside the beam tube in its any section the quantity of energy is constant. Hence it follows, that the exchange of energy between beam tubes does not occur.

Fermat's principle is considered to be fundamental for the geometrical optics. According to it, the optical length of the way in the given medium is less than the length of any other line, connecting two chosen points. The optical length of the way is meant as integral $\int_l ndl$ , where $l$ is the ray trajectory; $n = \sqrt{\varepsilon_a \mu_a / \varepsilon_0 \mu_0}$ is the refraction factor. Then, Fermat's principle

$$\int_l ndl = min \quad . \tag{1.6}$$

Hence it follows, that in homogeneous medium ( $n = const$ ) the ray trajectory is the straight line.

Laws of reflection and refraction in geometrical optics are received at fall of the flat electromagnetic wave on the flat boundary of two mediums. These laws are also correct for cases when radiuses of curvature of the medium's boundary and the wave front are longer than the wavelength. Thus, the incidence of the locally flat electromagnetic wave on the locally flat boundary is examined.

Laws of geometrical optics can be used when relative changes of dielectric and magnetic permittivity of medium, amplitudes of electric and magnetic field intensity on the distance of wavelength in the considered medium is less than

$$\frac{\Lambda \left| \mathrm{rot}\, \dot{\vec{E}}_m \right|}{\left| \vec{E}_m \right|} << 2\pi; \qquad \Lambda \left| \frac{grad \mu_a}{\mu_a} \right| << 4\pi;$$

$$\frac{\Lambda \left| \mathrm{rot}\, \dot{\vec{H}}_m \right|}{\left| \vec{H}_m \right|} << 2\pi; \qquad \Lambda \left| \frac{grad \varepsilon_a}{\varepsilon_a} \right| << 4\pi,$$

where $\Lambda$ is the wavelength of considered medium.

### 1.3. The equivalence principle

The field equivalence principle is the replacement of the actual sources of an electromagnetic field by a set of more convenient equivalent surface currents or charges.

Let $S$ be a closed surface, which divides space into regions $V_1$ and $V_2$ (Fig. 1.1). All field sources are situated in $S$ . If the character and arrangement of these sources are unknown, but vectors $E_S$ and $H_S$ on surface $S$ are known, the field in region $V_2$ , free from

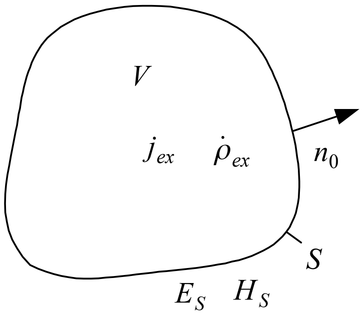

Fig. 1.1

sources, may be found from values $\dot{E}_S$ and $\dot{H}_S$. Thus, it is possible to proceed from the intensity of the field on surface $S$ to equivalent currents and charges.

The linear density of equivalent electric $j_S^e$ and magnetic $j_S^m$ currents is determined from expressions

$$j_S^e = \left[\vec{n}_0, \dot{H}_S\right]; \qquad (1.7)$$

$$j_S^m = -\left[\vec{n}_0, \dot{E}_S\right], \qquad (1.8)$$

where $n_0$ is the unit vector normal to $S$.

The surface density of equivalent electric $\dot{\sigma}_S^e$ and magnetic $\dot{\sigma}_S^m$ charges

$$\dot{\sigma}_S^e = \varepsilon_a \left(\dot{E}_S, \vec{n}_0\right); \qquad (1.9)$$

$$\dot{\sigma}_S^m = \mu_a \left(\dot{H}_S, \vec{n}_0\right). \qquad (1.10)$$

It is apparent from expressions (1.9), (1.10) that if tangential components of field are equal to zero, equivalent charges on surface $S$ are absent.

## 1.4. The duality principle

The duality principle follows from the symmetry of Maxwell's equations describing electric and magnetic fields. It essence is that the solution of Maxwell's equations for the electric field at given boundary conditions will be valid as well as for the magnetic field at the same boundary conditions, accepted for the magnetic field.

Thus, expressions describing the radiation field for the aerial with the magnetic current $I^m$ can be obtained from formulas for the radiation field of the aerial with electric current $I$ by the method of substitution

$$E \rightarrow H; \qquad I \rightarrow I^m; \qquad \varepsilon_a \leftrightarrow \mu_a;$$

$$H \rightarrow -E; \qquad I^m \rightarrow -I; \qquad W = 1/W. \qquad (1.11)$$

In this case vectors $E$ and $H$ should meet the suitable boundary conditions on identical interfaces.

### 1.5. Far-field radiation zone of an aerial

The antenna radiation field is divided into far, transition and near zones. Of particular interest is the field of the aerial at considerable distances, in the so-called far-field or the Fraunhofer zone, in which the field structure is stable and the amplitude of the field strength is proportional to the first order of distance.

Let us determine the radius of a far zone for aerials with the sizes larger than the wavelength (not point radiator). Let the aerial with the maximal linear size $L$ occupy volume $V$, in which the distribution of currents is given (Fig. 1.2). The origin of rectangular coordinate

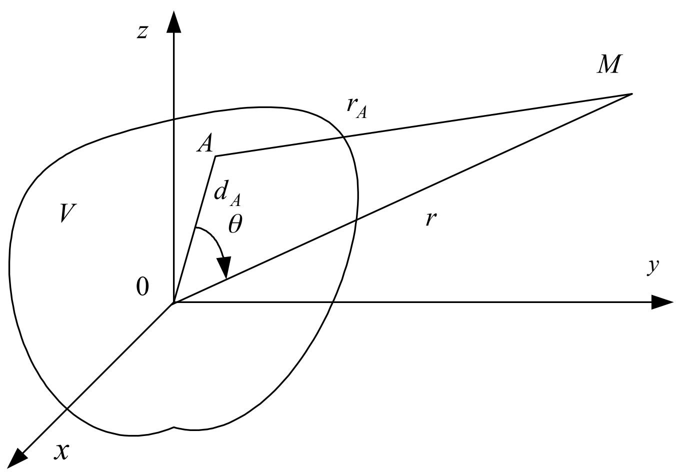


Fig. 1.2

system we will be so, that the maximal distance up to border of the aerial does not exceed $L/2$. Let us designate the point of observation removed enough from the aerial through M. We shall isolate two elements of current in volume $V$ : one – in the origin of coordinate system, another – in an arbitrary point $A$ .

Radiation fields of the sources located in these points, according to (1.4), will be characterized by functions of a spherical wave

$$\psi(r) = \frac{e^{-ikr}}{r}; \qquad \psi(r_A) = \frac{e^{-ikr_A}}{r_A},$$

where $r, r_A$ are distances from points **O** and $A$ to point $M$, respectively.

From Fig. 1.2

$$r_A = r - d,$$

or

$$r_A = \sqrt{r^2 + d^2 - 2dr\cos\theta}. \qquad (1.12)$$

As, for any value of vector $d$, the condition $d < r$ is satisfied, the expansion of expression (1.12) in a series on orders of the ratio is possible:

$$r_A = r\left[1 - \frac{d}{r}\cos\theta + \frac{1}{2}\left(\frac{d}{r}\right)^2 \sin^2\theta + \frac{1}{2}\left(\frac{d}{r}\right)^3 \sin^2\theta\cos\theta + \ldots\right]. \qquad (1.13)$$

When distance $r$ is of one order with size $L$, while determining the field intensity, it is necessary to use the exact value $r_A$ (1.12). The maximum distance, for which the application of the approximate formula (1.13) with the limited number of members results in significant errors in comparison with the exact formula (1.12), determines the radius of a near zone.

In an intermediate zone distance $r$ surpasses size $L$ so, that during decomposition of (1.13) on determining amplitudes of fields of elementary radiators (in this case in points 0 and $A$) one may neglect all members but the first one and consider that in functions of a spherical wave

$$\frac{1}{r_A} \approx \frac{1}{r}. \qquad (1.14)$$

In the phase factor ($e^{-ikr_A}$) such replacement is inadmissible, as it would result in significant phase errors, therefore, calculating phase angles the first three members of decomposition in equation (1.13) are taken:

$$r_A = r\left[1 - \frac{d}{r}\cos\theta + \frac{d^2}{2r^2}\sin^2\theta\right].$$

The minimal radius of an intermediate zone may be found from the inequality

$$\frac{0.5kL^3}{\left(8r_{\min}^2\right)} << 2\pi,$$

and maximal - from

$$\frac{0.5kL^2}{\left(4r_{\max}\right)} >> 2\pi,$$

where it is taken into account, that $d_{\max} = 0.5L$.

The intermediate zone is referred to as the Fresnel's zone.

In a far zone, which radius is even, when calculating the phase ratio, one can guess, that

$$r_A = r - d\cos\theta. \qquad (1.15)$$

In this case the phase error will be determined by the third member of series (1.13). The allowable error is the phase shift, which does not exceed $\pi/8$. The maximal phase shift is observed at $d = L/2$, therefore,

$$k\frac{L^2}{8r}\sin^2\theta \le \frac{\pi}{8} \qquad (1.16)$$

We shall obtain the minimal radius of the far zone from equation (1.16):

$$r_{\min} \ge 2\frac{L^2}{\lambda}. \qquad (1.17)$$

When studying the field in the far zone, which minimal radius satisfies condition (1.17), it is possible to assume, that rays which leave all points of the aerial and converge in a reference point, are parallel to each other, and the propagation difference of rays, which determines the phase ratio, can be found from formula (1.15). Calculating amplitudes of the field strength, which are radiated by aerial elements, one may assume that distances from elements of the aerial to a point of observation are equal to each other.

### 1.6. Intensity of radiation field of the aerial

Intensity of a radiation field of any aerial can be written as

$$\dot{E} = \vec{e}(\theta,\varphi)\dot{E}_{\max}F(\theta,\varphi)e^{i[\psi(\theta,\varphi)-kr]} \qquad (1.18)$$

or

$$\dot{E} = \vec{e}(\theta,\varphi)\dot{E}_{\max\,n}f(\theta,\varphi)e^{i[\psi(\theta,\varphi)-kr]}\,. \qquad (1.19)$$

In (1.18), (1.19) unit vector $e(\theta,\varphi)$ defines the position of vector $\dot{E}$ in space (polarization of wave); $\dot{E}_{\max}$, $\dot{E}_{\max\,i}$ is the amplitude of the field strength; $F(\theta,\varphi), f(\theta,\varphi)$ are functions of the field intensity, which depend on angles of the spherical coordinate system, at equality of $F(\theta,\varphi)$ maximum to unit and when maximum of $f(\theta,\varphi)$ may be of any value; $e^{i[\psi(\theta,\varphi)-kr]}$ is the phase factor, which is determined by dependence of phase on angles $\psi(\theta,\varphi)$ and phase incursion $kr$, where $k = 2\pi/\lambda$.

Similar expression can also be written for the magnetic component of the field. Thus, it is necessary to remember, that vectors $E$ and $H$ of the radiation field are mutually perpendicular, perpendicular to a propagation direction and change in phase in the lost-free medium. Their ratio is equal to the wave resistance of the medium

$$\frac{\dot{E}}{\dot{H}} = W\,.$$

### 1.7. Radiating power of aerial

The power of electromagnetic waves, radiated by the aerial, is usually known as the radiating power and designated through $P_{\Sigma}$. To determine the radiating power, let us arrange the aerial in the center of sphere $S$ of the sufficient radius $r$, chosen so that all points of the sphere surface are in a far zone. Let us find Poynting's vector on the surface of sphere $S$.

Complex Poynting's vector in each point of such sphere is

$$\dot{\vec{\Pi}} = \vec{r}_0\frac{1}{2}(\dot{E}\dot{H}^*) = \vec{r}_0\frac{1}{2W}(\dot{E}\dot{E}^*)\,, \qquad (1.20)$$

where $r_0$ is the ort of spherical coordinate system; $\dot{H}^*, \dot{E}^*$ are the conjugate values of the complex field intensity.

Using the period average value of Poynting's vector

$$\Pi_{av} = \mathrm{Re}\,\dot{\Pi}\,.$$

(1.21)

We may calculate the radiation power by the formula

$$P_{\Sigma} = \oint \Pi_{av} dS\,,$$

(1.22)

where $dS$ is the product of the surface element $dS$ on the unit vector $r_0$ normal to the surface.

The considered method is known as the method of Poynting's vector.

Let on the surface of sphere $S$ with radius $r$ the complex amplitude of the electric field intensity be equal to $\dot{E}(\theta,\varphi)$. Then, as it follows from expression (1.18), the module of the field intensity in any point is

$$\dot{E}(\theta,\varphi) = \dot{E}_{\max} F(\theta,\varphi)\,. \qquad (1.23)$$

The mean value of Poynting's vector according to formulas (1.21), (1.22) is

$$\vec{\Pi}_{av} = \vec{r}_0 \frac{E^2(\theta,\varphi)}{2W}\,. \qquad (1.24)$$

In the spherical coordinate system (Fig.1.3) the area of the surface element may be found as the product of the rectangle sides:

$$dS = r^2 \sin\theta d\theta\, d\varphi\,. \qquad (1.25)$$

The direction of vector $dS$ coincides with the direction of radius - vector $r$ .

Substituting expressions (1.24) and (1.25) in formula (1.22) we shall obtain:

$$P_{\Sigma} = \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} \frac{E^2(\theta,\varphi)}{2W} r^2 \sin\theta d\theta\, d\varphi\,. \qquad (1.26)$$

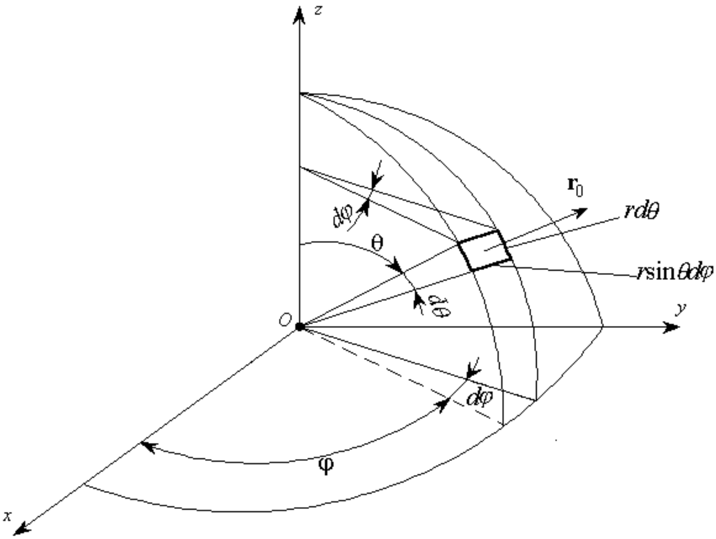

Fig. 1.3

Taking into account expression (1.25) we shall obtain:

$$P_{\Sigma} = \frac{E_{\max}^2 r^2}{2W} \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} F^2(\theta,\varphi) \sin\theta \, d\theta \, d\varphi \,. \qquad (1.27)$$

Formula (1.27) allows to calculate the radiation power at known field distribution in a far zone. Value of the radiation power enables us to determine some important parameters of the aerial.